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THIN-WALLED MEMBERS IN COMBINED TORSION AND FLEXURE

By Warner Lansing

STRUCTURAL DIVISION

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PAPERS

THIN-WALLED MEMBERS IN COMBINED TORSION AND FLEXURE

By Warner Lansing[1]

Synopsis

Stresses and deformations are investigated for thin-walled open section beams loaded in a manner that produces combined flexure and torsion. The analysis is based on the fundamental equations given by J. N. Goodier. It is shown that the expression for rotations of cross sections is analogous to the equation of equilibrium for a laterally loaded tie rod and because of the analogy, approximate solutions are available in important practical cases, the accuracy of which may be easily estimated.

The analysis is illustrated by considering briefly the behavior of channels loaded by transverse forces acting in the plane of the web. A comparison with test results shows reasonable agreement, and design procedures for braced channels have been suggested in agreement with this work.[2] In the rapidly expanding field of light-gage steel construction, members of this type are frequently used in a manner to which this analysis applies directly.

While lateral buckling problems are not stressed in this paper, it is incidentally shown that critical loads may be easily obtained as limiting cases of eccentric loading.

Introduction

The stability of thin-walled open section bars of arbitrary shape submitted to end thrust has been thoroughly investigated during the past several

Note.—Written comments are invited for publication; the last discussion should be submitted by **September 1, 1952.**

[1] Formerly instructor in Eng. Mechanics, Cornell Univ., Ithaca, N. Y.; Presently in Eng. Dept., Grumman Aircraft Eng. Corp., Bethpage, N. Y.

[2] "Performance of Laterally Loaded Channel Beams," by G. Winter, W. Lansing, and R. B. McCalley, *Engineering Structures Supplement*, Research, 1949.

decades.[3,4,5] The original theory has been extended[6,7] to include the more general problem of such bars under thrust, bending, and twisting. In all this work the determination of buckling loads has been emphasized.

There are many uses for thin-walled open sections bars, however, that result in definite rotational deflections and related secondary stresses from the very beginning of loading. Examples are the channel and the **Z**-section used as beams, that is, loaded by transverse forces in the plane of the web. Twist occurs from the start in the former case because the loads are not applied through the shear center and in the case of the **Z**-section because the loads are not acting in a principal plane. Although critical loads represent theoretical upper limits for the performance of such members, it is necessary in practical use to determine their stresses, rotations, and other design factors at subcritical loads. It is with this type of problem that this paper is primarily concerned.

The General Equations of Equilibrium

Considering a prismatical bar of open, but otherwise arbitrary, cross section, the general equations of equilibrium for loading by transverse forces may be written[7] as

$$M_\xi = -E I_x v'' = M_x + \beta M_y - u' M_z \quad (1a)$$

$$M_\eta = E I_y u'' = -\beta M_x + M_y - v' M_z \quad (1b)$$

$$M_\zeta = \left[G C + \frac{M_\xi K_1}{I_x} - \frac{M_\eta K_2}{I_y} \right] \beta' - E \Gamma \beta''' = u' M_x + v' M_y + M_z \quad (1c)$$

in which E is Young's modulus of elasticity; G is the shear modulus of elasticity; the location of the fixed coordinate system x, y, and z, with shear center at the origin and x and y parallel to the principal centroidal axes, is shown in Fig. 1; ξ, η, and ζ are the corresponding displaced cross-sectional axes; u and v are the displacements of the shear center of any section in the x and y directions, respectively; and β is the rotation of the section about the shear center. One, two, or more primes refer to the first, second, and so on, derivatives with respect to z.

The resultant of the force system acting on the portion of the member to the right of any section may be represented by a force vector applied at the displaced shear center of the section, plus a resultant moment. Then, following the right-hand screw convention, M_x, M_y, and M_z are the components of the

[3] "Torsion and Buckling of Open Sections," by H. Wagner, *Technical Memorandum No. 807*, National Advisory Committee for Aeronautics, Govt. Printing Office, Washington, D. C., October, 1936.

[4] "Torsion and Buckling of Open Sections," by H. Wagner and W. Pretschner, *Technical Memorandum No. 784*, National Advisory Committee for Aeronautics, Govt. Printing Office, Washington, D. C., January, 1936.

[5] "Twisting Failure of Centrally Loaded Open-Section Columns in the Elastic Range," by Robert Kappus, *Technical Memorandum No. 851*, National Advisory Committee for Aeronautics, Govt. Printing Office, Washington, D. C., March, 1938.

[6] "The Buckling of Compressed Bars by Torsion and Flexure," by J. N. Goodier, *Bulletin No. 27*, Cornell Univ. Eng. Experiment Station, Ithaca, N. Y., December, 1941.

[7] "Flexural-Torsional Buckling of Bars of Open Section," by J. N. Goodier *Bulletin No. 28*, Cornell Univ. Eng. Experiment Station, Ithaca, N. Y., January, 1942.

latter parallel to the x, y, and z axes, while M_ξ, M_η, and M_ζ represent the moment resolved into ξ, η, and ζ components.

As for the geometrical properties of the cross section, I_x and I_y are the usual principal moments of inertia, and C is the torsional constant. (For constant thickness t, C is very closely $\frac{1}{3} s\, t^3$, in which s is the developed length of the centerline.) Γ is the warping constant and its calculation is covered in standard texts on aircraft structural analysis[8,9,10] and will not be discussed here. (However, a formula for determining Γ for a channel section is given later.) Finally,

$$K_1 = \int_A (y - y_e)\,(x^2 + y^2)\, dA \quad \ldots\ldots\ldots\ldots\ldots\ldots (2a)$$

and

$$K_2 = \int_A (x - x_e)\,(x^2 + y^2)\, dA \quad \ldots\ldots\ldots\ldots\ldots\ldots (2b)$$

in which x_e and y_e are the coordinates of the centroid of the section, and A is its area. The terms in Eq. 1c involving K_1 and K_2 represent the contribution to the internal twisting moment of the ordinary bending fiber stress (σ_b) which may be written

$$\sigma_b = \frac{M_\xi\,(y - y_e)}{I_x} - \frac{M_\eta\,(x - x_e)}{I_y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The analogous effect in bars under uniform axial compression (P) results in the term—$P\ \rho^2\ \beta'$, in which ρ is the radius of gyration of the section taken about the shear center.

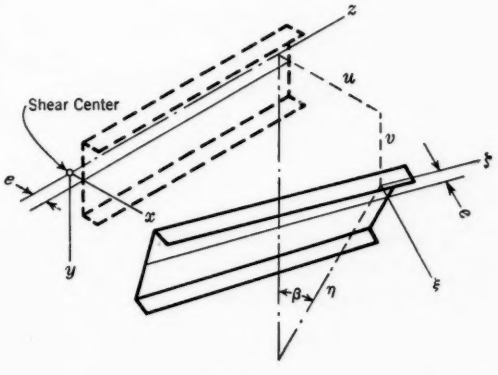


FIG. 1

In deriving Eq. 1 for use in stability problems, that is, for infinitesimal displacements, Mr. Goodier used first-order approximations for the direction cosines between x, y, z, and ξ, η, and ζ. It would appear that the expressions will also be sufficiently accurate for determining finite deformations and the accompanying stresses under stable conditions, provided the limitations on beam deflections of the ordinary engineering beam theory are observed; and rotations of cross sections are held to a maximum of, say, 5°.

[8] "Airplane Structural Analyses and Design," by Ernest E. Sechler and Louis G. Dunn, John Wiley & Sons, Inc., New York, N. Y., 1942.

[9] "Airplane Structures," by A. S. Niles and J. S. Newell, 3d Ed., John Wiley & Sons, Inc., New York, N. Y., Vol. 2, 1943.

[10] "Fundamentals of Aircraft Structures," by Millard V. Barton, Prentice-Hall, Inc., New York, N. Y., 1948.

The Tie Rod Analogy.—Eq. 1*c* may be written

$$E\,\Gamma\,\beta''' - T\,\beta' = -u'\,M_x - v'\,M_y - M_z \ldots\ldots\ldots\ldots (4)$$

in which $T = G\,C + M_\xi\,K_1/I_x - M_\eta\,K_2/I_y$. In this form, provided T is constant, Eq. 4 is mathematically identical to the equation of equilibrium of the laterally loaded tie rod. The warping rigidity of the thin-walled open section beam ($E\,\Gamma$) represents the flexural rigidity of the tie rod; β represents the deflection of the tie rod; T represents the tie rod's axial force; and the term $-u'\,M_x - v'\,M_y - M_z$ represents the shear in the tie rod caused by the transverse load. In the case of the tie rod, the following is a familiar technique for obtaining an approximate solution.[11] The problem is first solved with only the transverse loads acting, the axial force being set equal to zero. The deflections are then corrected by multiplying by a factor $\left[1 + \frac{T}{T_{cr}}\right]^{-1}$, in which T_{cr} is the Euler buckling load. Bending moments in the tie rod $E\,\Gamma\,\beta''$ are corrected by adding a term $T\,\beta$. If T is actually variable instead of constant, conservative estimates may be made for it.

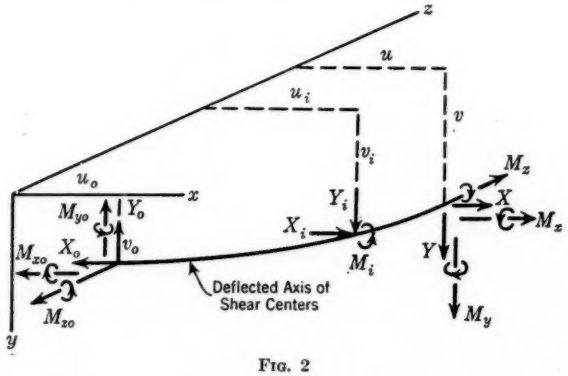


FIG. 2

In order to proceed along the foregoing lines in the case of thin-walled open section beams, T is set equal to zero, and Eq. 4 is then differentiated once with respect to z, yielding

$$E\,\Gamma\,\beta'^{v} = -u''\,M_x - u'\,M'_x - v''\,M_y - v'\,M'_y - M'_z \ldots\ldots (5)$$

Fig. 2 shows a portion of the deflected shear center axis of a beam, acted upon by various loads, concentrated or distributed as the case may be. It is seen that in the interval from z to $z + dz$, the increase in M_z is

$$-Y\,(u'\,dz) + X\,(v'\,dz) + m_z\,dz$$

in which m_z is the intensity of the moment of a distributed load about the shear center axis, if such is present, and X and Y are the components of the

[11] "Strength of Materials," by S. Timoshenko, 2d Ed., D. Van Nostrand Co., Inc., New York, N. Y., Vol. 2, 1941, p. 49.

ordinary beam shear in the x and y directions. Now,

$$M'_x = Y \quad (6a)$$

$$M'_y = -X \quad (6b)$$

and

$$M'_z = -u'Y + v'X + m_z \quad (6c)$$

Substituting in Eq. 5,

$$E\,\Gamma\,\beta'^v = -u''M_x - v''M_y - m_z \quad (7)$$

In what follows, it is assumed that it is permissible to neglect the terms in M_z in Eqs. 1a and 1b. This can be shown to be legitimate for I-beams, Z-sections, and channels loaded by transverse forces in the plane of the web.[12] The simplified expressions of Eqs. 1a and 1b may now be used to eliminate the terms u'' and v'' from Eq. 7, yielding

$$E\,\Gamma\,\beta'^v - \frac{I_x M^2_x + I_y M^2_y}{E\,I_x\,I_y}\beta = -\frac{M_x M_y (I_x - I_y)}{E\,I_x\,I_y} - m_z \quad (8)$$

In the case in which all the loads are initially applied in a plane that is parallel to the z axis, let M be the moment in that plane. Then $M_x = M\cos\theta$ and $M_y = M\sin\theta$, in which θ is the angle the moment vector M makes with the x axis, clockwise, being taken as positive when viewed in the positive z direction. New fixed axes are designated parallel with, and perpendicular to, the moment vector (M) by 1 and 2, respectively, such that when $\theta = 0$, axes 1 and 2 coincide with x and y. Then the moments and product of inertia of the cross section about the new axes are

$$I_1 = I_x\cos^2\theta + I_y\sin^2\theta \quad (9a)$$

$$I_2 = I_x\sin^2\theta + I_y\cos^2\theta \quad (9b)$$

$$I_{1,2} = (I_x - I_y)\sin\theta\cos\theta \quad (9c)$$

Here and throughout this paper, all section properties are referred to fixed axes, and are to be thought of as being calculated before the various sections are displaced by the applied loads.

With the substitutions from Eqs. 9, Eq. 8 becomes

$$E\,\Gamma\,\beta'^v - \frac{M^2 I_1}{E\,I_x\,I_y}\beta = -\frac{M^2 I_{1,2}}{E\,I_x\,I_y} - m_z \quad (10)$$

It will be noted that for sections with point symmetry, $K_1 = K_2 = 0$, and T in Eq. 4 is a constant, namely, $G\,C$. Under these conditions T could have been used explicitly in the subsequent work with no additional complications, yielding an additional term $-G\,C\,\beta''$ on the left of Eq. 10. With the latter included in Eq. 10, the equations of equilibrium for the lateral

[12] "Stresses in Thin Walled Open Section Beams Due to Combined Torsion and Flexure," by W. Lansing, thesis presented to Cornell Univ., at Ithaca, N. Y., in 1949, in partial fulfilment for the degree of Doctor of Philosophy.

stability of I-beams obtained by S. Timoshenko[13] are contained as special cases.

If M is constant, such as in loading by end couples, the solution of Eq. 10 presents no problem. On the other hand, if M varies with z, as it will for loading by transverse forces, an exact solution is very difficult, and approximate methods must be employed.

The homogeneous equation associated with Eq. 10, that is,

$$E\,\Gamma\,\beta'^{v} - \frac{M^2\,I_1}{E\,I_x\,I_y}\,\beta = 0 \qquad (11)$$

is the well-known differential equation for the whirling speed of a rotating shaft. The iteration method of A. Stodola,[14] used in connection with this problem, is thus suggested.

Example: Channel Simply Supported at the Ends, Loaded at the Middle

Deflection Analysis.—For the specific case of a simply supported channel of length $2\,l$, and depth $2\,h$, subject to a single concentrated load $2\,P$ applied at midspan at the top of the web, $\theta = 0$ and $M = P\,(l - z)$, $m_z = 0$. The origin is at midspan. Under these conditions Eq. 10 reduces to

$$E\,\Gamma\,\beta'^{v} - \frac{P^2\,l^2}{E\,I_y}\left(1 - \frac{z}{l}\right)^2\beta = 0 \qquad (12)$$

The boundary conditions are

$$\beta(l) = \beta'(0) = \beta''(l) = 0 \qquad (13a)$$

and

$$\beta'''(0) = \frac{P\,[e + h\,\beta(0)]}{E\,\Gamma} \qquad (13b)$$

The indicated value of the term $\beta'''(0)$ is obtained by using the left and middle expressions of Eq. 1*c* and noting that at $z = 0$, $\beta' = 0$, and $M_\zeta = -P\,[e + h\,\beta(0)]$. In the latter, the term $\sin\beta(0)$ has been replaced by the term $\beta(0)$ and e is the distance from the shear center to the web.

Following the method of Mr. Stodola, an expression satisfying the boundary conditions for β, but arbitrary otherwise, should now be substituted in Eq. 12:

$$\beta_1 = \beta_1(0)\left[1 - \frac{6}{5}\frac{z^2}{l^2} + \frac{1}{5}\frac{z^4}{l^4}\right] \qquad (14)$$

This equation has the appropriate general shape of a single half-wave, and the fact that it does not satisfy one of the boundary conditions, namely, Eq. 13*b*, is of no practical importance. Now,

$$\frac{E^2\,\Gamma\,I_y}{P^3\,l^2}\,\beta'^{v}{}_2 = \beta_1(0)\left[1 - \frac{6}{5}\frac{z^2}{l^2} + \frac{1}{5}\frac{z^4}{l^4}\right]\left(1 + \frac{z}{l}\right)^2 \qquad (15)$$

[13] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1936, Chapter 5, pp. 239–286.

[14] "Mathematical Methods in Engineering," by Theodore von Kármán and Maurice A. Biot, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1940.

The subscripts 1 and 2 are to identify, respectively, the assumed and derived functions for β.

By direct integration,

$$\frac{E^2\,\Gamma\,I_y}{P^2\,l^3}\,\beta'''_2 = \beta_1(0)\left[\frac{z}{l} - \frac{z^2}{l^2} - \frac{1}{15}\frac{z^3}{l^3} + \frac{3}{5}\frac{z^4}{l^4} - \frac{1}{5}\frac{z^5}{l^5} - \frac{1}{15}\frac{z^6}{l^6} + \frac{1}{35}\frac{z^7}{l^7}\right] + C_1 \quad (16)$$

in which C_1 is the constant of integration. From Eq. 13*b*, it can be seen that

$$C_1 = \frac{E\,I_y\,h}{P\,l^3}\,\beta_2(0) + \frac{E\,I_y\,e}{P\,l^3} \quad (17)$$

Integrating a second time and using the third boundary condition of Eq. 13*a* yields

$$\frac{E^2\,\Gamma\,I_y}{P^2\,l^4}\,\beta''_2 = \beta_1(0)\left[-\frac{323}{1400} + \frac{1}{2}\frac{z^2}{l^2} - \frac{1}{3}\frac{z^3}{l^3} - \frac{1}{60}\frac{z^4}{l^4} + \frac{3}{25}\frac{z^5}{l^5} - \frac{1}{30}\frac{z^6}{l^6} - \frac{1}{105}\frac{z^7}{l^7} + \frac{1}{280}\frac{z^8}{l^8}\right] + \left[\frac{E\,I_y\,h}{P\,l^3}\,\beta_2(0) + \frac{E\,I_y\,e}{P\,l^3}\right]\left[\frac{z}{l} - 1\right] \quad (18)$$

In a similar way, two more integrations give

$$\frac{E^2\,\Gamma\,I_y}{P^2\,l^6}\,\beta_2 = \beta_1(0)\left[\frac{6709}{79{,}600} - \frac{323}{2800}\frac{z^2}{l^2} + \frac{1}{24}\frac{z^4}{l^4} - \frac{1}{60}\frac{z^5}{l^5} - \frac{1}{1800}\frac{z^6}{l^6} + \frac{1}{350}\frac{z^7}{l^7} - \frac{1}{1680}\frac{z^8}{l^8} - \frac{1}{7560}\frac{z^9}{l^9} + \frac{1}{25{,}200}\frac{z^{10}}{l^{10}}\right] + \left[\frac{E\,I_y\,h}{P\,l^3}\,\beta_2(0) + \frac{E\,I_y\,e}{P\,l^3}\right]\left[\frac{1}{3} - \frac{1}{2}\frac{z^2}{l^2} + \frac{1}{6}\frac{z^3}{l^3}\right] \quad (19)$$

Before proceeding further, it is desirable to introduce the maximum fiber stress caused by elementary beam action, $\bar{\sigma} = P\,l\,h/I_x$. Then, the three dimensionless constants of Eq. 19 involving sectional properties can be expressed as

$$\frac{E^2\,\Gamma\,I_y}{P^2\,l^6} = \frac{I_y\,\Gamma\,h^2\,E^2}{I^2_x\,l^4\,\bar{\sigma}^2} = \frac{1}{\alpha} \quad (20a)$$

$$\frac{E\,I_y\,h}{P\,l^3} = \frac{h^2\,I_y\,E}{l^2\,I_x\,\bar{\sigma}} = \gamma \quad (20b)$$

and

$$\frac{E\,I_y\,e}{P\,l^3} = \frac{h\,e\,I_y\,E}{l^2\,I_x\,\bar{\sigma}} = \delta \quad (20c)$$

From the discussion of the tie rod analogy, it is understood that for the conjugate tie rod, the right-hand side of Eq. 15 is proportional to a distributed lateral load. In order to obtain the approximately correct rotations for this

problem, accordingly the right-hand side of Eq. 19 is multiplied by the factor

$$\nu = \left[1 + \frac{T}{T_{cr}}\right]^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which T is as defined previously. In the resulting equation, $\beta_1(0)$ is set equal to $\beta_2(0)$; dropping the subscripts and employing Eq. 20, at midspan ($z = 0$)

$$\frac{1}{\alpha}\beta(0) = \nu\left\{\beta(0)\frac{6709}{75{,}600} + \beta(0)\frac{\gamma}{3} + \frac{\delta}{3}\right\} \quad \ldots\ldots\ldots\ldots (22)$$

Solving for $\beta(0)$ gives

$$\beta(0) = \frac{\nu\,\alpha\,\frac{\delta}{3}}{1 - \nu\,\alpha\left[\frac{6709}{75{,}600} + \frac{\gamma}{3}\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The iterative process need be carried no further, as this expression is already sufficiently accurate for engineering use.

Limiting Case of Eccentric Loading.—Eq. 23 is also valid for the rotation at midspan of a simply supported I-beam loaded by a central force at the top flange at a distance e from the plane of the web. The point of application of the force may be shifted to the centroid by allowing e and h in Eq. 13*b* to shrink to zero. This in turn requires that γ and δ go to zero in Eq. 23. Thus $\beta(0)$ can be different from zero only if

$$1 - \nu\,\alpha\,\frac{6709}{75{,}600} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

An exact solution of this lateral buckling problem for I-beams has been obtained by Mr. Timoshenko,[13] and thus a comparison will give an estimate of the accuracy of the approximate method.

Referring to the discussion on the tie rod analogy, T_{cr} for this problem is the "Euler load" associated with the equation $E\,\Gamma\,\beta''' + G\,C\,\beta' = 0$. The boundary conditions given in Eq. 13, with the change previously discussed (that is, $\beta'''(0) = 0$) are equivalent to those of a simply supported column of length $2\,l$. Hence,

$$T_{cr} = \frac{\pi^2\,E\,\Gamma}{4\,l^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

and

$$\nu = \left[1 + \frac{G\,C\,4\,l^2}{E\,\Gamma\,\pi^2}\right]^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Using Eq. 26 and the expression for α from Eq. 20*a*, and letting $P = Q/2$, $l = L/2$, and $a^2 = G\,C/E\,\Gamma$, Eq. 24 may be solved for Q_{cr} to give

$$Q_{cr} = \frac{m\,\sqrt{E\,I_y\,G\,C}}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which $m = 17.11\sqrt{1 + \frac{\pi^2}{a^2\,L^2}}$.

Written in this form the equation is identical to the Timoshenko solution. Comparing m for the same range of values of $a^2 L^2$ as given by Mr. Timoshenko[15] the difference was never found to exceed 0.5%. The approximate solution is thus satisfactory in practical applications.

Stress Distribution in Loaded Channel.—Returning to the channel problem, the following simplified way of visualizing the stress distribution was indicated by George Winter, M. ASCE, in an unpublished research report. The displacement of the midspan section may be considered as proceeding in the successive stages depicted in Fig. 3. The section, then, is thought of as being first displaced downward in simple translation. The stresses induced would be those of the ordinary beam theory and are indicated in character by the appropriate signs at the corners of the section, minus signs denoting compression and plus signs tension.

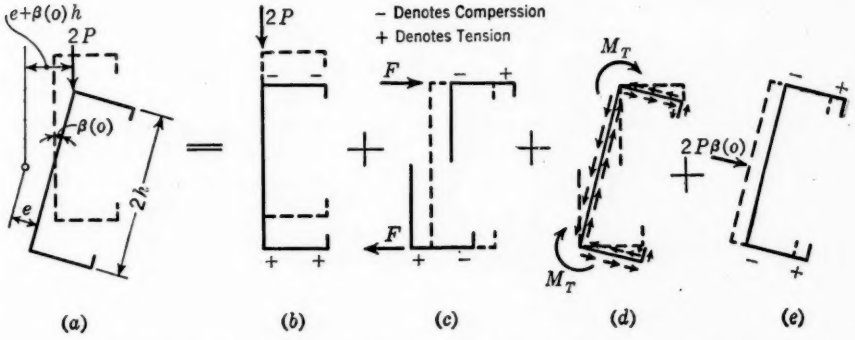


FIG. 3.—STRESS DISTRIBUTION IN A CHANNEL SUBJECTED TO TORSION AND FLEXURE

Next, the channel is considered as cut and the two halves displaced much like two individual beams, resulting in the appropriate indicated corner stresses. To fit the two halves together, they are next rotated about their individual shear centers, giving rise only to shear stresses of the ordinary St. Venant character. In this inclined position, finally, the component of the vertical load parallel to the major axis causes additional bending about the minor axis, with its corresponding normal stresses. This picture is not an exact one but is discussed only to indicate the general type of the resulting stress distribution; it is shown in another paper[2] that this simplified concept, somewhat modified, leads directly to an entirely satisfactory approximate analysis for design purposes. It is evident that under such a stress distribution, cross sections distort out of their original planes. For this reason, the stresses associated, in particular, with the second displacement stage (Fig. 3(*c*)) are generally known as warping stresses.

The stresses of the first and last displacement stages (Fig. 3(*b*) and 3(*e*)) are of course represented by σ_b, Eq. 3. A more precise definition of the warping

[15] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., New York, N. Y., 1936, p. 267, Table 22.

stresses is provided by the expression,

$$\sigma_z = E\,\beta''\,(w_1 - \bar{w}_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

first stated by H. Wagner[3,4] for thin-walled open section bars in nonuniform torsion. The quantity $w_1 - \bar{w}_1$ is the warping displacement, for unit twist, measured from a plane normal to the ζ axis and containing the centroid of the warped surface of the cross section. The term $w_1 - \bar{w}_1$ is a section property; Fig. 4 shows its variation along the mean line of a channel with lips. The ordinates of the points in Fig. 4 are defined as follows:

$$W_1 = h\,e \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29a)$$

$$W_2 = h\,(e - b) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29b)$$

$$W_3 = h\,(e - b) - (e + b)\,d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29c)$$

The formula for the warping constant Γ in this case is

$$\Gamma = \frac{2\,t}{3}\left[(h + e)\,W^2{}_1 + (b - e)\,W^2{}_2 + d\,(W^2{}_2 + W_2\,W_3 + W^2{}_3)\right] \ldots (30)$$

At any section the stress σ_z is seen to be distributed in a manner proportional to $w_1 - \bar{w}_1$. Comparing Fig. 4 with the stress distribution that accompanies the second displacement stage, Fig. 3(*c*), the two are seen to be rather similar, the difference consisting in linear variations of stress in the web and lips according to the Wagner theory, as contrasted with the constant values of the approximate method. A slight modification of the latter accounts satisfactorily for this difference. The conclusion, reached by a study of Fig. 3, that the maximum fiber stress occurs in compression at the upper junction of web and flange, is thus confirmed by the more exact theory. Evaluating σ_b and σ_z at this point and adding,

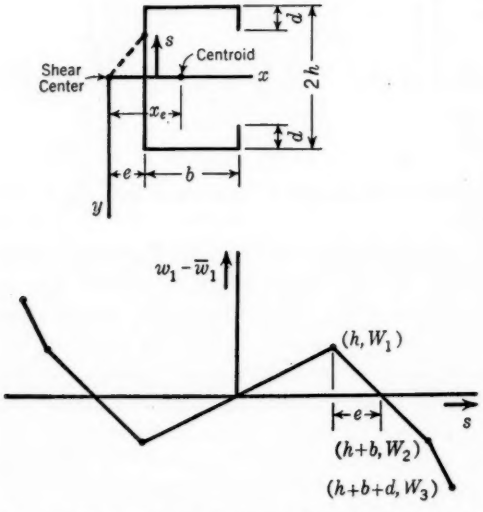


FIG. 4.—VARIATION OF WARPING DISPLACEMENT ALONG THE MEAN LINE OF A CHANNEL

$$\sigma_{\max} = -\,\bar{\sigma}\left(1 + \frac{S_x}{S_y}\,\beta(0)\right) + E\,\beta''(0)\,W_1 \quad \ldots\ldots\ldots\ldots (31)$$

in which $S_x = I_x/h$, $S_y = I_y/(x_e - e)$ ($x_e - e$ is the distance from the midplane of the web to the centroid), and W_1 is the value of $w_1 - \bar{w}_1$ for the point

in question, as given in Fig. 4. In Eq. 23 $\beta(0)$ is expressed and $\beta''(0)$ is obtained by setting $z = 0$ in Eq. 18 and adding a term $\frac{T}{E\,\Gamma}\beta(0)$. The analogous step in tie rod problems consists of adding to the term $E\,\Gamma\,\beta''$ (the "bending moment" due to transverse loads only) the "moment of the axial load," $T\,\beta$, as discussed previously. Thus,

$$\beta''(0) = -\frac{\alpha}{l^2}\left[\delta + \beta(0)\left(\frac{323}{1400} + \gamma\right)\right] + \beta(0)\frac{T}{E\,\Gamma} \quad \ldots\ldots(32)$$

In this problem as $M_\eta = M\,\beta$, T may be written as

$$T = G\,C\left[1 - \frac{I_x\,K_2}{I_y\,h}\frac{\bar{\sigma}}{G\,C}\left(1 - \frac{z}{l}\right)\beta\right] \quad \ldots\ldots(33)$$

(K_1 vanishes for reasons of symmetry.) It is seen that the value of T varies along the bar, having a maximum value at the supports

$$T_{max} = G\,C \quad \ldots\ldots(34a)$$

and a minimum value at midspan

$$T_{min} = G\,C\left(1 - \frac{I_x\,K_2}{I_y\,h}\frac{\bar{\sigma}}{G\,C}\beta(0)\right) \quad \ldots\ldots(34b)$$

Two calculations are made, in each of which T is assumed to be constant. These will serve as upper and lower bounds for the true answer.

Results of Numerical Analyses.—Analyses have been made for four channel sections that appear to represent the extremes for the commercially available shapes as commonly listed.[16] The dimensions of these sections are given in Table 1 referred to the typical section in Fig. 5. The results for the most unfavorable of these sections, namely, No. 3, are shown in Fig. 5. The usual criterion for design purposes would stipulate that the carrying capacity of the member is reached when yielding starts in the most highly stressed fiber. For a beam with no tendency to twist, this means, as usual, that the maximum fiber stress $\left(\bar{\sigma} = \frac{P\,l\,h}{I_x}\right)$ becomes equal to the yield point. In a channel, however, this fiber stress corresponds only to the first of the four displacements (Fig. 3(*b*)) and is augmented at the most highly stressed point (upper left corner) by the additional stresses represented by the $\beta(0)$ and $\beta''(0)$ terms on the right-hand side of Eq. 31.

TABLE 1.—DIMENSIONS IN INCHES OF ANALYZED CHANNEL SECTIONS

Section No.	Dimension (Fig. 5)				
	D	*B*	*d*	*t*	*R*
1	12.0	3.50	1.0	0.135	3/16
2	12.0	3.50	0.7	0.075	3/32
3	3.0	1.75	0.7	0.105	3/16
4	3.0	1.75	0.4	0.048	3/32

[16] "Light Cage Steel Design Manual," American Iron and Steel Inst., New York, N. Y., 1949.

To assess the reduced efficiency of a channel section as compared to a similar section prevented from twisting, it is simplest to plot against the span length that simple bending stress $\bar{\sigma} = \frac{P\,l\,h}{I_x}$ that will result in yield stress at the most unfavorable fiber. This information has been plotted in Fig. 5 for a yield point of 33,000 lb per sq in. for mild structural steel ($E = 30 \times 10^6$ lb per sq in.). In the same figure is shown the angle of rotation $\beta(0)$ that exists when yielding begins in the upper left corner.

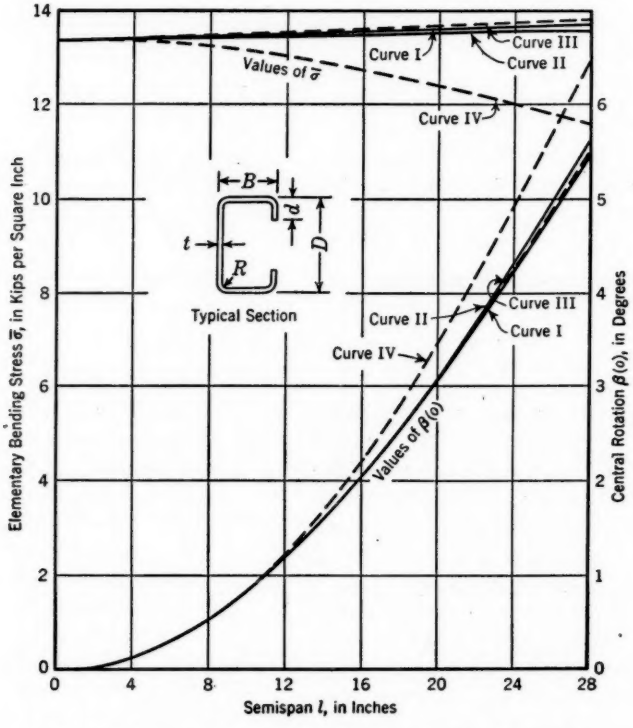


FIG. 5.—ANALYSIS RESULTS FOR CHANNEL SECTION NUMBER 3

The curves designated I are based upon an assumed constant value of $T = G\,C$, and $T_{cr} = \frac{\pi^2\,E\,\Gamma}{4\,l^2}$. This would be quite proper if the problem at hand were a question of lateral stability. In that case $\beta(0)$ would be very small, and from Eq. 34 $T_{max} = T_{min} = G\,C$. The problem presently being investigated is different in that the eccentrically applied central force causes rotation from the very beginning of loading. Thus for $\beta(0)$ equal to a maximum value of, say, 5°, $\bar{\sigma}$ from curve I is 13,800 lb per sq in. and using these figures plus $G = 11.5 \times 10^6$ lb per sq in. (mild steel), T_{min} is 0.89 T_{max}. The term T being analogous to the axial force in tie rods, which of course tends to

reduce deflections and bending moments, the curves marked *I* are seen to be unconservative, and may be considered an upper bound for the correct curves. A lower bound may be obtained based upon a T equal to the $T_{\min}$ of Eq. 34*b*, into which values of $\bar{\sigma}$ and $\beta(0)$ are substituted from curves I.

Curves designated II have been plotted on this basis. The two bounds are sufficiently close together that the position between them of the curves representing the exact solution is immaterial for engineering purposes. Indeed, for the semispan $l = 28$ in., for which $\beta(0)$ is approximately $5\frac{1}{2}°$ by either of the two curves, the ordinates to the two $\bar{\sigma}$ curves differ by less than 2%, and for decreasing semispan, the spread shrinks to zero. This would indicate that it is permissible to ignore the K_2 term in Eq. 33 for T, in the case of channels loaded in the plane of the web. It will be appreciated, however, that this statement does not hold for types of loading that result in substantial bending about the secondary axis of the channel.

An additional conclusion may be drawn from Fig. 5. It is seen that even for impracticably short spans, the load that causes incipient yielding in the unbraced channel is less than half the load that would initiate yielding if twisting were prevented by appropriate bracing. This is in sharp contrast to the performance of unbraced I- and Z-sections that for very short spans, can be subjected to a moment M such that $\bar{\sigma} = M\frac{h}{I}$ = yield point, and in which $\bar{\sigma}$ must be reduced only in larger spans, because of buckling for I-beams and of twisting for Z-sections. The reason for this difference is the fact that the location of the shear center behind the web in channels results in a primary twisting moment; in I- and Z-sections, on the other hand, the shear center and centroid are coincident, so that torsional moments are merely caused by the rotation of cross sections and the deflection of the axis of shear centers out of a plane parallel to the loading plane.

This conclusion leads to the examination of the possibility of another simplification in the analysis for determining β. The large primary twisting moment induced by the eccentric shear center location suggests that the β term in Eq. 10, although of the essence in stability problems, may be negligible here. Omission of this term reduces Eq. 10 to

$$E\,\Gamma\,\beta'^{v} = -\,m_z \qquad (35)$$

and using $T = G\,C$, yields expressions that plot as the dashed curves III of Fig. 5. These should be compared with curves I, which are different only in that the β term has been included. Since the curves practically coincide, this approximation is clearly permissible.

It may be somewhat surprising to note that the efficiency of this channel increases slightly with increasing span. It would seem more natural to observe a decrease in efficiency caused by the secondary effects associated with the central rotation. This and certain other effects can best be explained by a qualitative argument based upon Fig. 3. Consider the channel to be loaded by a force $2\,P$ that varies inversely with the span $2\,l$, so that the elementary bending stress $\bar{\sigma}$ remains constant. Under these conditions the principal part

of the torsional moment, $P\,e$, varies inversely as l. Then the central rotation angle $\beta(0)$, if it were dependent only upon the torsional rigidity of the channel (the action depicted in Fig. 3(d)), would be of constant magnitude $\frac{P\,e\,l}{G\,C}$.

On the other hand, if the deformation were to depend only on the action of Fig. 3(c) an entirely different situation would prevail. Then, each half of the section performs as if it were a simple beam acted upon by the force

$$F = 2\,P\,[e + \beta(0)\,h]\frac{1}{2\,h} \qquad (36)$$

Since the location of the load F on this fictitious horizontal beam, and its span, are identical to those for the channel as a whole (for vertical bending), the resulting horizontal bending moments and the corresponding stresses shown in Fig. 3(c) are directly proportional to those caused by vertical bending, Fig. 3(b) except for the minor influence of $\beta(0)$. The central deflection of the half-beam, $\delta = \dfrac{F\,(2\,l)^3}{48\,E\left(\dfrac{I_y}{2}\right)}$, is related to the central rotation $\beta(0)$ by the formula $\beta(0) = \delta/h$ and again neglecting secondary effects, $\beta(0) = \dfrac{P\,e\,l^3}{3\,E\,I_y\,h^2}$. For this action then, the central rotation would vary as the square of the span, $P\,l$ being constant.

It follows from the two expressions for $\beta(0)$ that for a given section, for very short spans, the channel is much stiffer in differential bending (Fig. 3(c)) than it is in the St. Venant torsion (Fig. 3(d)), although, the span is made sufficiently large, the opposite situation is true. In the case of section 3 (the stockiest of the sections of Table 1), for which the curves of Fig. 5 are plotted, as the semispan l goes from 0 to 28 in., the performance passes from the differential bending regime into an intermediate one in which an appreciable part of the torsional load is carried by the St. Venant torsion. Thus, as l increases, more of the total stress σ_{max} is available as primary bending stress $\bar{\sigma}$. The nonlinearity of the torsional moment because of the influence of $\beta(0)$ diminishes this effect, but does not quite cancel it.

In the cases of the other three channels listed in Table 1, the performance is predominantly in the differential bending regime, and the $\bar{\sigma}$ curves drop off with increasing span. As an example of the behavior of such relatively thinner sections, Fig. 6 gives the corresponding information for section 1.

In order to evaluate this effect quantitatively, the dashed lines designated curve IV in Fig. 5 are plotted. They are based upon Eq. 35, and differ from curve III only in that the tie rod correction is omitted; that is, the value of T is taken equal to zero. This is equivalent to assuming the torsional rigidity, $G\,C$, to be zero. For the section of Fig. 5, the results are rather conservative when compared with curves III. In the case of each of the other three sections, the curves III and IV all lie much closer together, and have spreads between the $\bar{\sigma}$ curves of less than $4\frac{1}{2}\%$ for $\beta(0)$ equal to approximately 5° (see Fig. 6).

Whether or not the torsional rigidity plays a significant role may be determined by the size of the parameter $\nu = \left[1 + \frac{G\,C}{E\,\Gamma}\frac{4\,l^2}{\pi^2}\right]^{-1}$. The ratio $\frac{G\,C}{E\,\Gamma}\frac{4\,l^2}{\pi^2}$ is a measure of the relative stiffnesses of the channel in the St. Venant torsion and in differential bending. This follows from the two expressions previously given for $\beta(0)$, when it is understood that $I_y\,h^2$ is merely an approximation

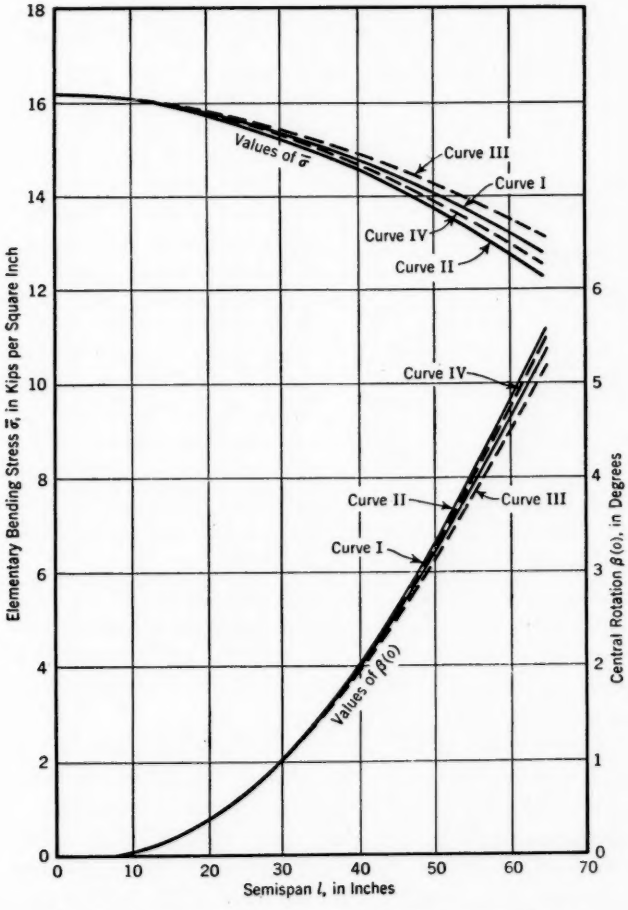


FIG. 6.—ANALYSIS RESULTS FOR CHANNEL SECTION NUMBER 1

for Γ. Indeed, the two quantities are identical in the case of an I-section. Evaluating ν for spans such that $\beta(0) = 5°$ from curves I, $\nu = 0.920$, 0.973, 0.734, and 0.927 for sections 1 through 4, respectively. It is seen that for $0.9 < \nu < 1$, the effect of the $G\,C$ term on $\beta(0)$ is very minor. For smaller values of ν, such as 0.734 for section 3, the increase in carrying capacity because of the torsional rigidity becomes significant.

Braced Channels

Solution for Intermediate Bracing.—The action of intermediate bracing is now easily visualized. It prevents horizontal displacement of the fictitious half-beams at the points of bracing; consequently these half-beams are converted from simple beams of span length equal to that of the entire channel to continuous beams with individual spans equal to the distance between braces. If, for example, the braces were applied at the third points of the span, the half-beam would perform as the continuous beam shown in Fig. 7 when loaded by the force F as given by Eq. 36. The resulting maximum horizontal bending moment on the half-beam and the corresponding stresses of Fig. 3(*b*) are less than one quarter of those obtained without bracing, as can be verified easily by continuous beam analysis.

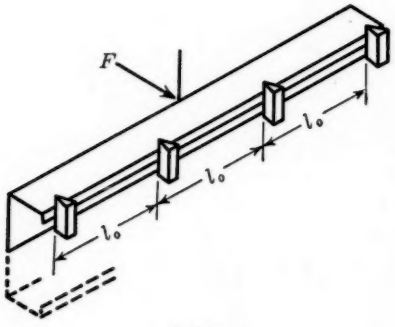


Fig. 7

The tie rod correction for braced channels can always be neglected without noticeable error because of the reduced equivalent span. For example, in the case of bracing at the third points discussed above, the ratio T/T_{cr} is one ninth of the value without braces. A reduction of this magnitude will certainly move ν extremely close to unity.

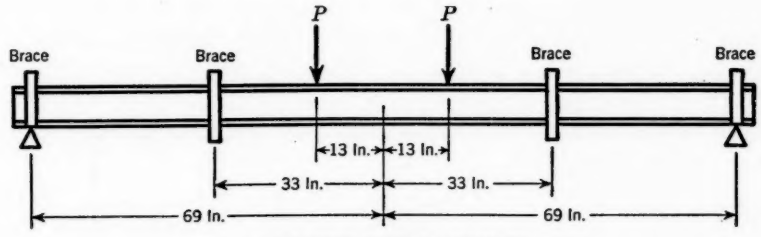
The procedure, then, in braced channel problems, is based upon Eq. 35. Drawing upon the analogy between it and the equation for an ordinary beam, solutions are effected by any of the usual beam theory methods. Warping stresses are thereafter obtained from Eq. 28.[2]

Tests.—Tests on braced channels have been performed by Mr. Winter and R. B. McCalley (Jun. M. ASCE) as one phase of an extensive research project on light-gage steel structures sponsored at Cornell University (Ithaca, N. Y.) by the American Iron and Steel Institute.[2] A brief summary of some of the results is presented here for completeness.

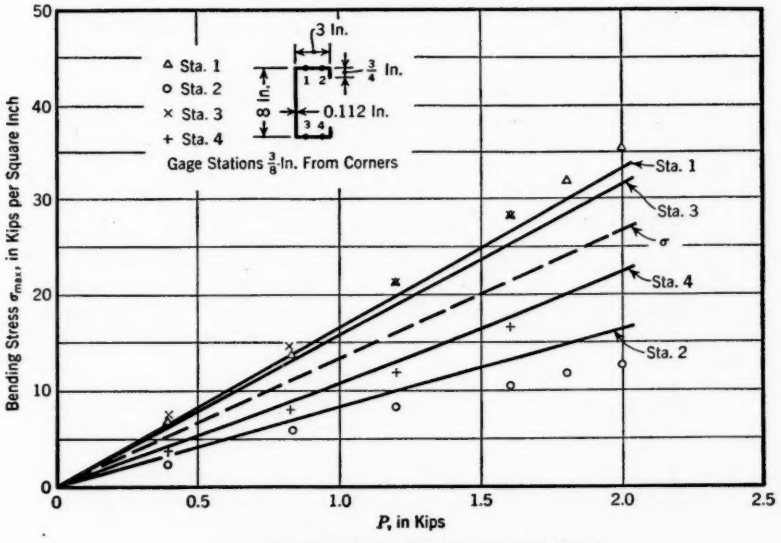
Seven different types of thin-walled channels were tested. Their depths ranged from 4 to 8 in., the widths from 2.5 to 4 in., and the thicknesses from 0.060 to 0.151, while the lips were about $\frac{3}{4}$ in. for all sections. Loads were applied through multiple ball bearings and knife edges to allow lateral and rotational motion as free from frictional constraint as was possible to achieve in a hydraulic testing machine. Analyses were made for the channels braced as in Fig. 8(*a*).

The measured and computed stresses are shown in Fig. 8(*b*) for that channel for which the agreement between experimental and theoretical values was least satisfactory. It is seen that the points of highest stress are the junctures of web and flange, with the stress being slightly larger at the upper one of these two points. These maximum stresses will govern practical design. The theoretical values are seen to agree with those from tests within about 5%.

Agreement is seen to be quite satisfactory also for station 4 in Fig. 8(*b*) but is rather far off for station 2. It should be noted that in thin-walled members loaded in flexure, the cross sections distort out of their original shape to a degree that depends upon the width and thickness of the flanges and the depth



(*a*) BRACING OF TEST CHANNEL



(*b*) MEASURED AND COMPUTED STRESSES FOR LEAST SATISFACTORY TEST CHANNEL

FIG. 8.—ANALYSIS OF BRACED CHANNEL

of the member.[17,18] This distortion, though small in practical terms, is likely to affect the distribution of the longitudinal stresses over the section, a factor that is not accounted for in the analysis.

[17] "Stress Distribution in and Equivalent Width of Flanges of Wide, Thin Wall Steel Beams," by G. Winter, *Technical Note No. 784*, National Advisory Committee for Aeronautics, Govt. Printing Office, Washington, D. C., November, 1940.

[18] "Performance of Thin Steel Compression Flanges," by George Winter, Third Congress of the International Assn. for Bridge and Structural Eng. Preliminary Publications, Liège, France, September, 1948, p. 137.

Simply Supported Z-Beams

Eq. 10 has also been solved for simply supported **Z**-beams loaded in the plane of the web.[12] The particular loads considered were end couples, central force, and uniformly distributed load. Eq. 14 was again used for β, leading to the same type of calculations as described herein for the channel.

The counterpart of curves I of Fig. 6 is given in Fig. 9 for the **Z**-section that has the same cross-sectional dimensions as those of channel section No. 1, except that the flanges are turned in opposite directions. However, the loading

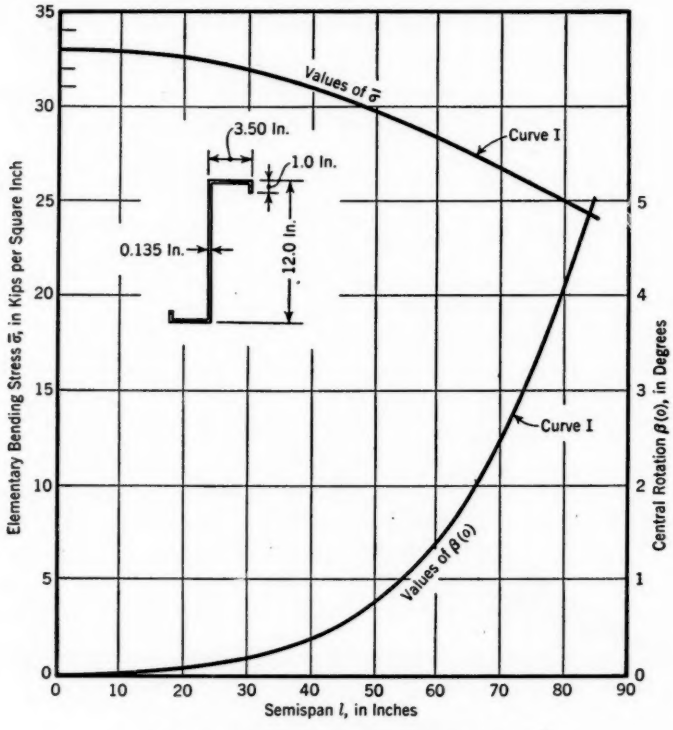


Fig. 9.—Analysis Results for Z-Section

is by end couples, as this results in larger secondary stresses and deflections than either of the other two types of loading systems for which analyses were made. The maximum stress again occurs at the upper junction of web and flange, although the stress at the extremity of the upper flange lip is only slightly smaller when $\beta(0)$ is of the order of 5°. It is seen that for very small spans the **Z**-section may be subjected to a moment (M) such that $\bar{\sigma} = \dfrac{M\,h\,I_2}{I_x\,I_y}$ = yield point. As the semispan l increases, $\bar{\sigma}$ drops off, and for $l = 85$ in. (for which $\beta(0)$ is 5°) $\bar{\sigma} = 24{,}100$ lb per sq in., or about 0.73 σ_{max}.

In order to determine whether the β term in Eq. 10 may be neglected in **Z**-beam problems, that equation is best written as

$$E\,\Gamma\,\beta'^{v} = \frac{M^2\,I_1}{E\,I_x\,I_y}\left(\frac{I_{12}}{I_1} - \beta\right) - m_z \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

In the case of a **Z**-section loaded by end couples, with the origin at midspan, $\beta(l) = \beta'(0) = \beta''(l) = \beta'''(0) = 0$, $m_z = 0$, and clearly, β can be replaced by zero only if it is small compared with I_{12}/I_1. Since the latter ranges from -0.218 to -0.614 for the sections available commercially,[16] this step is not permissible when $\beta(0)$ is of the order of 5° (0.0873 radians), that is, for unbraced **Z**-sections acting at reasonable spans. However, it is easily seen that when the **Z**-section is braced against rotation at several intervals, the rotation will be very small throughout. For example, for the **Z**-section of Fig. 9, braced at the third points, the central rotation is reduced to less than 1% of its former value. For braced **Z**-sections then, the formula,

$$E\,\Gamma\,\beta'^{v} = \frac{M^2\,I_{12}}{E\,I_x\,I_y} - m_z \ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

may be used with good accuracy. The statements regarding the tie rod correction for channels hold equally well for **Z**-sections.

Acknowledgments

The writer is indebted to Mr. Winter, Head, Department of Structural Engineering, Cornell University, for suggesting the thesis topic on which this paper is based and for helpful criticism during the course of the work. Gratitude is also expressed for the assistance of Mr. McCalley, Research Associate at Cornell University, in the numerical calculations.

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122	Apr., 1952	"Engineering Aspects of Diffraction and Refraction," by J. W. Johnson	Sept. 1

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